International Journal of Management and Humanities (IJMH)
ISSN: 2394–0913 (Online), Volume-5 Issue-4, December 2020

Transient Solution of $M^X/M/C$ Queueing Model for Homogenous Servers, with Catastrophes, Balking & Vacation

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Abstract: ***In this paper we analyze $M^X/M/C$ Queueing model of homogenous service rate with catastrophes, balking and vacation. Here we consider the customers, where arrival follow a poisson and the service follows an exponential distribution. Based on the above considerations, under catastrophes, balking and vacation by using probability generating function along with the Bessel properties we obtain the transient solution of the model in a simple way.***

Keywords: Homogeneous servers, vacation, balking, catastrophes, Bessel function.

I. INTRODUCTION

Queueing system can be classified as customer arriving, waiting for service and leaving the system after being served. More contributions have been developed in different directions of queueing theory. Ammar [1] has discussed the transient behavior of a multiple vacations queue with impatient customers. Bharathidass et al [3] describe the single server of bulk service queue with the concepts like vacation, breakdown and repair and also, he explained about the expected number of units in the system under various positions of the server. Doshi [5] has developed the queueing system with two types of vacation named as primary and secondary also he gave an overview of some results and his methodology used to obtain various results for two vacation models. MaragathaSundari and Srinivasan [6]analyze a M/G/1 queue with single server vacation of Bernoulli schedule.

Pazhani et al [7] analyze a single server queueing system of infinite capacity along with second optional service. Sasikala and Indhira[9] explain about an overview of bulk service with queueing models. Dharmaraja [4] found that the transient solution for markovian queueing model with heterogeneous server and catastrophes. Ayyappan and Sathiya[2]declare that the bulk arrival queue with two types of service, vacation in the transient model.Sherbiny [10] describes the varying arrival and departure rate with an infinite server queue in the transient model. Based on the above literature the transient solution for bulk customers entering the system under catastrophes, balking and vacation effects are analyzed.

II. MODEL DESCRIPTION

In this paper we consider a queueing model for$M^X/M/C$ homogenous servers wherethe service rate is μ with exponential distribution. Customers arriving at the system follows a poisson distribution with rate λ.The system follows First- come, First -served (FCFS) basis and let $P_n(t)$ denotes the probability that there are n customers in the system. Suppose the system may suffer with catastrophes ε. The impatient customers followa distribution rate β.Moreover, the homogenous server goes for a vacation at the rate of θ. Additionally this model having i customers at initially.

The governing equations are

$$\frac{dP_{i0}(t)}{dt} = \mu P_{i1}(t) - (\lambda_0 + \lambda_1 + \theta + \varepsilon)P_{i0}(t) + \varepsilon \quad (1)$$

$$\frac{dP_{in}(t)}{dt} = \lambda_0 P_{i0}(t) + (\lambda_1 + \theta)P_{i,n-1}(t) - (\lambda_1 + \varepsilon + n\mu + \theta)P_{in}(t) + (n+1)\mu P_{i,n+1}(t) \quad 1 \le n < C \quad (2)$$

$$\frac{dP_{ic}(t)}{dt} = (\lambda_1 + \theta)P_{ic-1}(t) - (\lambda_2\beta + \varepsilon + c\mu + \theta)P_{ic}(t) + c\mu P_{i,c+1}(t) \quad n = c \quad (3)$$

$$\frac{dP_{in}(t)}{dt} = (\lambda_2\beta + \theta)P_{in-1}(t) - (\lambda_2\beta + \varepsilon + c\mu + \theta)P_{in}(t) + c\mu P_{i,n+1}(t) \quad ,n > c \quad (4)$$

By defining $P(z,t) = q_c(t) + \sum_{n=1}^{\infty} P_{n+c}(t)z^n$ (5)

$$P(z,0) = 1 \ and \ q_c(t) = \sum_{n=0}^{c} P_{in}(t) \quad (6)$$

Adding eqn (1) – (3) we get

$$\frac{dq_c(t)}{dt} = -(\lambda_2\,\beta + \theta)P_{ic}(t) + c\mu P_{i,c+1}(t) - \varepsilon q_c(t) + \varepsilon \quad (7)$$

Revised Manuscript Received on December 15, 2020.
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Now multiplying the equation by z^n summing over the corresponding limit values, we get,

$$\frac{d\sum_{n=1}^{\infty}P_{i,n+c}(t)z^n}{dt}=(\lambda_2\beta+\theta)zP_{ic}(t)\left[-(\lambda_2\beta+\varepsilon+c\mu+\theta)+\left[(\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right]\right]\sum_{n=1}^{\infty}P_{i,c+n}(t)z^n-c\mu P_{i,c+1}(t) \quad (8)$$

Adding eqn (7) & (8) using (5) we get

$$\frac{dP(z,t)}{dt}=\left[\left((\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right)-(\lambda_2\beta+\varepsilon+c\mu+\theta)\right]P(z,t)-[\left((\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right)-(\lambda_2\beta+c\mu+\theta)]q_c(t)+\varepsilon+(\lambda_2\beta+\theta)(z-1)P_{i,c}(t) \quad (9)$$

By Solving the above first order differential equation we get,

$$P(z,t)=e^{\left[\left((\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right)-(\lambda_2\beta+\varepsilon+c\mu+\theta)\right]t}+\int_0^t[(\lambda_2\beta+\theta)(z-1)P_{ic}(u)-[\left((\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right)-(\lambda_2\beta+c\mu+\theta))]q_c(u)\,e^{\left[\left((\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right)-(\lambda_2\beta+\varepsilon+c\mu+\theta)\right](t-u)}du+\varepsilon\int_0^t e^{\left[\left((\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right)-(\lambda_2\beta+\varepsilon+c\mu+\theta)\right](t-u)}du \quad (10)$$

By considering, $a=2\sqrt{(\lambda_2\beta+\theta)c\mu}$ & $b=\sqrt{\frac{(\lambda_2\beta+\theta)}{c\mu}}$ then we obtain

$$e^{\left((\lambda_2\beta+\theta)z+\frac{c\mu}{z}\right)t}=\sum_{n=-\infty}^{\infty}(bz)^nI_n(at) \quad (11)$$

By Applying eqn (11) in eqn (10) we get

$$P(z,t)=e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)t}\left[\sum_{n=-\infty}^{\infty}(bz)^nI_n(at)\right]+(\lambda_2\beta+\theta)\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}\left[\sum_{n=-\infty}^{\infty}(bz)^n[b^{-1}I_{n-1}a(t-u)-I_na(t-u)]P_{ic}(u)\right]du+\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}\sum_{n=-\infty}^{\infty}(bz)^n[-(\lambda_2\beta+\theta)b^{-1}I_{n-1}a(t-u)+(\lambda_2\beta+\theta+c\mu)I_na(t-u)-c\mu bI_{n+1}a(t-u)]\,q_c(u)du+\varepsilon\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}\sum_{n=-\infty}^{\infty}(bz)^nI_na(t-u)du \quad (12)$$

By comparing the coefficient of z^non both the side of eqn (12) we obtain for n=1,2… by using eqn (5)

$$P_{in+c}(t)=e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)t}b^nI_n(at)+(\lambda_2\beta+\theta)\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}[b^{n-1}I_{n-1}a(t-u)-b^nI_na(t-u)]P_{ic}(u)du-\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}[(\lambda_2\beta+\theta)b^{n-1}I_{n-1}a(t-u)-(\lambda_2\beta+\theta+c\mu)b^nI_na(t-u)+c\mu b^{n+1}I_{n+1}a(t-u)]q_c(u)du+\varepsilon\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}b^nI_na(t-u)du \quad (13)$$

Again, comparing the constant terms in eqn (12) and substituting n=0 we get

$$q_c(t)=e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)t}I_0(at)+(\lambda_2\beta+\theta)\int_0^t e^{-(\lambda_2\beta+\varepsilon+\theta+c\mu)(t-u)}[b^{-1}I_1a(t-u)-I_0a(t-u)]P_{ic}(u)du+\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}[-aI_1a(t-u)+(\lambda_2\beta+c\mu+\theta)I_0a(t-u)]q_c(u)du+\varepsilon\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}I_0a(t-u)du \quad (14)$$

Eqn (13) implies that

$$\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}[(\lambda_2\beta+\theta)I_{n+1}a(t-u)b^{n-1}-(\lambda_2\beta+\theta+c\mu)b^nI_na(t-u)+c\mu b^{n+1}I_{n-1}a(t-u)]q_c(u)du=e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)t}b^nI_n(at)$$

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$+(\lambda_2\beta+\theta)\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}\,[b^{n-1}I_{n+1}a(t-u)-b^nI_na(t-u)]P_{ic}(u)du+\varepsilon\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}b^nI_na(t-u)du$ (15)

By using (15) in eqn (13) we get $P_{in}(t)$ implies that

$$P_{in+c}(t)=nb^n\int_0^t e^{-(\lambda_2\beta+\varepsilon+c\mu+\theta)(t-u)}\left[\frac{I_na(t-u)}{(t-u)}\right]P_{ic}(u)du \quad (16)$$

The remaining probabilities $P_{in}(t)$ $n=0,1,2\ldots c$ can be found by solving the system of equations (1)& (2).

The matrix form of eqn (1) & (2) can be represented as

$$\frac{dP(t)}{dt}=AP(t)+c\mu P_{ic}(t)e_1+\varepsilon e_2 \quad (17)$$

where the matrix $A=(b_{ij})_{(c\times c)}$ is given as

$$\begin{bmatrix}-(\lambda_0+\lambda_1+\theta+\varepsilon) & \mu & \cdots & 0\\ (\lambda_0+\lambda_1+\theta) & -(\lambda_1+\varepsilon+\mu+\theta) & & 0\\ \vdots & \vdots & & \vdots\\ 0 & 0 & & (c-1)\mu\\ 0 & 0 & & -(\lambda_1+\varepsilon+(c-1)\mu+\theta\end{bmatrix}$$

$P(t)=\left(P_0(t),P_1(t)\ldots P_{c-1}(t)\right)^T$ $e_1=(0,0\ldots1)^T$ and $e_2=(1,0\ldots0)^T$ are column vectors of order C. $P^*(s)$ denote the laplace transforms of $P(t)$.

By taking the laplace transforms of eqn (17) and solving of $P^*(s)$, we get

$$P^*(s)=\left[c\mu P_{ic}{}^*(s)e_1+P(0)+\frac{\varepsilon}{s}e_2\right][SI-A]^{-1} \quad (18)$$

Thus $e^TP^*(s)+P_{ic}{}^*(s)=q_c{}^*(s)$ (19)

By define $f(s)=[(s+\lambda_2\beta+c\mu+\varepsilon+\theta)-\sqrt{(s+\lambda_2\beta+c\mu+\varepsilon+\theta)^2-a^2}]$

Taking laplace of eqn (14) and solving for $q_c{}^*(s)$ we get,

$$s(s+\varepsilon)q_c{}^*(s)=(s+\varepsilon)+sP_{ic}{}^*(s)\frac{1}{2}[f(s)-ab] \quad (20)$$

By using eqn (20) in eqn (19) we get,

$$P_{ic}{}^*(s)=\frac{(s+\varepsilon)}{s}\left(\frac{1-se^T[SI-A]^{-1}\left(P(0)+\frac{\varepsilon}{s}e_2\right)}{(s+\varepsilon+(\lambda_2\beta+\theta))-\frac{1}{2}f(s)+c\mu e^T[SI-A]^{-1}e_1(s+\varepsilon)}\right) \quad (21)$$

Based on Raju and Bhat [8] by letting $[SI-A]^{-1}=(d_{ij}{}^*(s))_{c\times c}$

Here $[SI-A]^{-1}$is almost lower triangular matrix, and $d_{ij}{}^*(s)$ is given by

$$d_{ij}{}^*(s)=\begin{cases}\dfrac{1}{(j+1)\mu}\quad\dfrac{u_{c,j+1}(s)u_{i,0}(s)-u_{i,j+1}(s)u_{c,0}(s)}{u_{c,0}(s)} & ,j=0,1,\ldots c-2\\ \dfrac{u_{i,0}(s)}{u_{c,0}(s)} & j=c-1.\end{cases} \quad (22)$$

$u_{i,j}(s)$ are given as,

$u_{i,i}=1$ $\qquad i=0,1\ldots c-1.$

$u_{i+1,i}=\dfrac{s+(\lambda_2\beta+\theta)+\varepsilon+i\mu}{(i+1)\mu}$ $\qquad i=0,1\ldots c-2.$

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$$u_{i+1,i-j} = \frac{(s+(\lambda_2\beta+\theta)+\varepsilon+i\mu)u_{i,i-j}-(\lambda_2\beta+\theta)u_{i-1,i-j}}{(i+1)\mu}, \qquad j \le i, i = 1,2 \dots c-2.$$

$$u_{c,j} = \begin{cases}(s+(\lambda_2\beta+\theta)+\varepsilon+(c-1)\mu)u_{c-1,j}-(\lambda_2\beta+\theta)u_{c-2,j} & j = 0,1 \dots c-2 \\ (s+(\lambda_2\beta+\theta)+\varepsilon+(c-1)\mu & j = c-1\end{cases}$$

(23)

For other i and j $u_{i,j} = 0$

Thus Eqn (21) becomes $P_{ic}{}^*(s) = \frac{(s+\varepsilon)}{s}\left(\frac{(1-(s+\varepsilon)\sum_{i=0}^{c-1} b_{i0}{}^*(s)}{[(s+\varepsilon+(\lambda_2\beta+\theta))-\frac{1}{2}f(s)+c\mu(s+\varepsilon)\sum_{j=0}^{c-1} b_{j,c-1}{}^*(s)]}\right)$

(24)

and Eqn (18) yields,

$$P_k{}^*(s) = \left(1+\frac{\varepsilon}{s}\right)b_{k,0}{}^*(s) + c\mu b_{k,c-1}{}^*(s)P_{ic}{}^*(s) \qquad (25)$$

where $e_1 = (0,0 \dots 1)^T$ $\quad$ $and\ e_2 = (1,0 \dots 0)^T$

$$P_{ic}{}^*(s) = \frac{(1+\frac{\varepsilon}{s})E^*(s)}{\frac{1}{2}f(s)\left[1-\frac{2c\mu(1-F^*(s))}{(s+(\lambda_2\beta+\theta)+c\mu+\varepsilon)-\sqrt{(s+(\lambda_2\beta+\theta)+c\mu+\varepsilon)^2-a^2}}\right]}$$

(26)

$$E^*(s) = \sum_{i=0}^{c-1}\frac{E_i}{s-s_i} \qquad (27)$$

and $F^*(s) = \sum_{i=0}^{c-1}\frac{F_i}{s-s_i}$ (28)

Here $E_i = \lim_{s\to s_i}(s-s_i)[1-\sum_{l=0}^{c-1}(s+\varepsilon)b_{lo}{}^*(s)$ (29)

$$F_i = \lim_{s\to s_i} s-s_i[\sum_{l=0}^{c-1}(s+\varepsilon)b_{l,c-1}{}^*(s)\,] \qquad (30)$$

Further Eqn (26) becomes

$$P_{ic}{}^*(s) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}(n+1)\frac{(-1)^m}{c\mu}\left(\frac{a}{2(\lambda_2\beta+\theta)}\right)^{n+1}\binom{n}{k}$$

$$\left[\left(1+\frac{\varepsilon}{s}\right)E^*(s)(F^*(s))^m\frac{[(s+(\lambda_2\beta+\theta)+c\mu+\varepsilon)-\sqrt{(s+(\lambda_2\beta+\theta)+c\mu+\varepsilon)^2-a^2}]^{n+1}}{(n+1)a^{n+1}}\right] \qquad (31)$$

By taking Laplace inverse of eqn(31) we obtain

$$P_{ic}(t) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{(-1)^m}{c\mu}\left(\frac{a}{2(\lambda_2\beta+\theta)}\right)^{n+1}\binom{n}{k}$$

$$\left[\int_0^t E(t-u)\int_0^u F^{c(m)}(u-v)e^{-((\lambda_2\beta+\theta)+c\mu+\varepsilon)v}\frac{I_{n+1}(av)}{v}dudv\right]$$

$$+\varepsilon\left[\int_0^t G(t-u)\int_0^u F^{c(m)}(u-v)e^{-((\lambda_2\beta+\theta)+c\mu+\varepsilon)v}\frac{I_{n+1}(av)}{v}dudv\right] \qquad (32)$$

Here $G(t) = \int_0^t E(u)du\ and\ F^{c(m)}$is m-fold convolution of $F(t)$ with $F^{c(0)} = \delta(t)$ is a direct delta function.

Moreover, Laplace inverse of eqn (25) yields

$$P_k(t) = b_{k0}(t) + \varepsilon\int_0^t b_{k0}(u)\,du + c\mu\int_0^t b_{kc-1}(u)\,P_{ic}(t-u)du,$$

Retrieval Number:100.1/ijmh.L03250831219
DOI:10.35940/ijmh.L0325.125420
Journal Website: www.ijmh.org

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$$k = 0,1 \ldots c-1 \tag{33}$$

Thus eqns (16), (32) & (33) are the state probability values of our model.

III. CONCLUSION

In this paper by using probability generating function technique the time dependent solution of the homogenous$M^X/M/C$ queueing model with catastrophes, balking and vacationis derived. In future, with the use of transient state probability, the various performance measures are also analyzed.

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Kavitha.G, (Research Scholar) is a Maths teacher and has finished M.Phil in Mathematics. Also finished B.Ed in Mathematics and finished Diploma in Operations Research, Published journals in the field of Transient Analysis Queueing Model.